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ABSTRACT

Hydroelectric dams emit greenhouse gases, especially methane (CH\textsubscript{4}), which is produced by decomposition of organic matter under anoxic conditions at the bottom of the reservoirs. A part of this gas is released by bubbling and diffusion through the surface of the reservoir, and part from the water that passes through the turbines and spillways. The portion of the emission that occurs through the reservoir surface has been calculated in estimates by the Brazilian government agency responsible for energy planning. The official calculation used a power law that resulted in an estimate of CH\textsubscript{4} 76\% lower than a corrected “basic” mean that represents the arithmetic mean of the measurements that served as the basis of the calculation. Expressed in comparison to the official value, the approximation of the arithmetic mean is 320\% higher. We show that the adjustment that was applied in the official estimates was based on several mathematical errors, and that the real value should be higher, rather than lower, than the arithmetic mean. We compared various possible methods for generating a “corrected” estimate, all with results far above the official values. A best method was identified that indicates a reservoir-surface emission 345\% higher than the official value. For Brazil’s 33×10\textsuperscript{3} km\textsuperscript{2} of reservoirs, the total impact of the underestimate of surface emissions of CH\textsubscript{4} is almost as large as the emission produced by burning fossil fuels in greater São Paulo, while the total emission of the reservoir surfaces surpasses the emission of this city. Emissions from the water that passes through the turbines and spillways represent an additional impact on global warming.

Key-words: Brazil, Carbon, Dams, Global warming, Greenhouse effect, Methane.

INTRODUCTION

Greenhouse gas emissions from reservoirs, especially methane (CH\textsubscript{4}), were identified first by Rudd et al. (1993) in Canada. Since then, a series of studies has estimated emissions, with a wide variety of results and interpretations (see review in Fearnside 2008; see also Ramos et al. 2006; DelSontro et al. 2010). Dos Santos et al. (2008) argued that, although greenhouse gas emissions by hydroelectric plants exist, their magnitude is substantially lower than the emissions indicated by various estimates in the literature (e.g., Fearnside 2002, Kemenes et al. 2007). The estimates given by dos Santos et al. (2008) are the same as those used in the publication of ELETROBRÁS (2000) on the total methane emissions of surfaces of reservoirs in the country. The arguments of dos Santos et al. (2008), and their overall conclusion that emissions are small, have been challenged over the course of an extensive debate (e.g., Fearnside 2004, 2006, 2008, 2009). However, a part of the reasoning of dos Santos et al. (2008) still requires further investigation. This is a calculation that these authors present of emissions by bubbling and diffusion through the surface of reservoirs. Dos Santos et al. (2008) used the frequency distribution of different magnitudes of emissions (in mg m\textsuperscript{-2} d\textsuperscript{-1} = kg km\textsuperscript{-2} d\textsuperscript{-1}) in existing measures, based on measurements at seven Brazilian reservoirs (Miranda, Três Marias, Barra Bonita, Segredo, Xingó, Samuel and Tucuruí). These authors applied the calculation using a power law to
adjust the estimated average emission **downward** in relation to the value indicated by a simple mean of the measured data. Here, we identify a series of errors in the calculations of dos Santos et al. (2008), and show that the adjustment should be **upward**, as compared to the simple mean. This implies that there is greater impact of greenhouse-gas emissions from reservoirs.

A phenomenon that is characterized by many small events and a few large events, such as earthquakes and forest fires, can be better represented by the power law than by the arithmetic mean of the observations. This is because any sampling period will be insufficient to capture the rare events of very large magnitude. The difficulty inherent in observing very large values logically implies that the value estimated by a power law must be greater than the arithmetic mean, the magnitude of this difference depending on the frequency distribution of events of different sizes for each phenomenon. The logic of a power law, therefore, leads to an adjustment of the values in the opposite direction from the adjustment made by dos Santos et al. (2008) and ELETROBRÁS (2000) for emissions from hydroelectric dams.

**THE CALCULATION OF DOS SANTOS ET AL. (2008): PROBLEMS AND ALTERNATIVES**

According to dos Santos et al. (2008), use of the arithmetic mean overestimates emissions from hydroelectric reservoirs surfaces. However, this conclusion is based on mathematical errors. The justification for the assertions of dos Santos et al. (2008) is contained in the thesis of dos Santos (2000), and is repeated in ELETROBRÁS (2000). These studies assume that the set of emissions values measured at different times and locations follows a power law. For this distribution, the probability density \( f \) of an emission measure of \( I \ mg \ m^{-2} \ d^{-1} \) will be:

\[
f(I) \propto I^{-\lambda}
\]

where the exponent \( \lambda \) is a constant ("\( \propto \)" means "proportional to"). Dos Santos et al. (2008) assumed values for a lower bound \( I_{\text{min}} \) and a upper bound \( I_{\text{max}} \) for the distribution. Since any probability distribution must comply with

\[
\int_{I_{\text{min}}}^{I_{\text{max}}} f(I) dI = 1
\]  

(1),

it follows that

\[
f(I) = \left[ \frac{\lambda - 1}{I_{\text{min}}^{\lambda-1} - I_{\text{max}}^{\lambda-1}} \right] I^{-\lambda}
\]  

(2)

(for \( \lambda \neq 1 \); Pueyo 2007). The theoretical mean for a distribution is defined as:

\[
\bar{T} = \int_{I_{\text{min}}}^{I_{\text{max}}} I f(I) dI
\]  

(3).

In the case of a power law, from Eqs. 2 and 3:

\[
\bar{T} = \left( \frac{\lambda - 1}{2 - \lambda} \right) \left( \frac{I_{\text{max}}^{2-\lambda} - I_{\text{min}}^{2-\lambda}}{I_{\text{min}}^{\lambda-1} - I_{\text{max}}^{\lambda-1}} \right)
\]  

(4).
Dos Santos (2000) followed another route and reached a different result. Instead of using the probability density \( f(I) \), he used an expected number of occurrences \( N(I) \) for each \( I \):

\[
N(I) = AI^{-\lambda}
\]

where \( A \) is a constant.

This formalism would not present a problem if it were used correctly. Given that \( N(I) \) is proportional to \( f(I) \), it follows from Eqs. 1, 3 and 5 that:

\[
I_{\text{max}} \int N(I) \, dI \quad \text{and} \quad \int N(I) \, dI
\]

However, dos Santos (2000) used a different expression:

\[
\overline{I}_{\text{Santos1}} = \frac{\int_{I_{\text{min}}}^{I_{\text{max}}} IN(I) \, dN(I)}{\int_{I_{\text{min}}}^{I_{\text{max}}} N(I) \, dN(I)}
\]  

(6).

This is the first mistake: by definition, the mean of the emissions of gases results from integrating over the emission values \( I \), not over the frequency values \( N \).

The result of Eq. 6 is:

\[
\overline{I}_{\text{Santos1}} = \frac{2\lambda (I_{\text{min}}^{-2\lambda} - I_{\text{max}}^{-2\lambda})}{(2\lambda - 1)(I_{\text{min}}^{-2\lambda} - I_{\text{max}}^{-2\lambda})}
\]

(7).

Dos Santos (2000) considered that \( I_{\text{max}}^{-2\lambda} \to 0 \) and \( I_{\text{max}}^{-2\lambda} \to 0 \), obtaining

\[
\overline{I}_{\text{Santos1}} = \frac{2\lambda}{2\lambda - 1} I_{\text{min}}
\]

(8).

The elimination of \( I_{\text{max}} \) is acceptable in Eq. 7 (assuming that \( \lambda > 0.5 \) and \( I_{\text{max}}/I_{\text{min}} \) has a very large value), while the term with \( I_{\text{min}} \) has great weight in the truest expression of the mean, Eq. 4 (except for when \( \lambda > 2 \) and \( I_{\text{max}}/I_{\text{min}} \) is "large"; the values of \( \lambda \) estimated by dos Santos (2000) were smaller).

The second problem is that, when making the calculations, dos Santos (2000) introduced the values of \( \lambda \) with the wrong sign. Thus, the author implicitly applied the equation:

\[
\overline{I}_{\text{Santos2}} = \frac{2(-\lambda)}{2(-\lambda) - 1} I_{\text{min}}
\]

that is,
Neither Eq. 8 found by dos Santos (2000) nor Eq. 9 (which he used implicitly) resembles the real theoretical mean in Eq. 4. So what is the relationship between the true theoretical mean (Eq. 1) and the sample mean? If one attributes to \( I_{\text{max}} \) the value of the largest emission observed, the means calculated by the two methods must coincide, except for the inevitable sampling error, without any significant bias. Moreover, this error affects both methods, since to applying Eq. 4 one must estimate the parameters of the distribution. However, by increasing the sample size one also increases the largest value observed, which, in turn, underestimates the theoretical maximum. This would not be a problem for calculating the mean of other statistical distributions, but it is a problem for the power law (Pueyo 2007), where the most extreme events represent a very important part of total emissions. The consequence is that, in most cases, the sample mean underestimates the real average, unlike the conclusion of dos Santos (2000), ELETROBRÁS (2000) and dos Santos et al. (2008).

Knowing the correct equations, the power law can be applied to the emissions data, but still there are other issues that should be considered.

Dos Santos (2000) estimated \( \lambda \) with a simple regression on the histogram with logarithmic axes. This method results in a very large bias (Pueyo & Jovani 2006). Some more effective procedures are: (1) applying the regression to the data grouped into multiplicative bins (Pueyo & Jovani 2006), (2) obtaining the maximum likelihood estimator (MLE, see Box I) (White et al. 2008), or (3) Bayesian methods.

The definition of the bounds \( I_{\text{min}} \) and \( I_{\text{max}} \) is also not trivial. Dos Santos (2000) used the midpoint of the first and the last bins of the histogram. According to this criterion, values depend on the width of the bins, which was chosen arbitrarily by the author. In the case of \( I_{\text{min}} \), a small modification produces a major change in the estimated mean: note that in Eq. 4, \( \bar{I} \rightarrow 0 \) when \( I_{\text{min}} \rightarrow 0 \) (for \( \lambda > 1 \)). The range of values \([I_{\text{min}}, I_{\text{max}}]\) where the power law is considered valid must be based on data, not an arbitrary criterion.

Finally, there is the problem of values that fall outside the range of validity of the power law, which were disregarded by dos Santos (2000). In this distribution, the greatest concentration is at the extreme lower end \( I_{\text{min}} \), but \( I_{\text{min}} \) is always greater than zero. Probably there are emissions data in the range \( 0 \leq I < I_{\text{min}} \). These data are also relevant for calculating the mean. There are two methods to incorporate them. The first is to examine the non-power-law part \((0 \leq I < I_{\text{min}})\) and the power-law part \((I_{\text{min}} \leq I \leq I_{\text{max}})\) separately. The second is to use a single expression that represents a good approximation to the two parts. For example, Ramos et al. (2006) used the generalized Pareto law:

\[
\bar{I}_{\text{Santos}2} = \frac{2\lambda}{2\lambda + 1} I_{\text{min}}
\]  

(9).
\[ f(I) = \sigma^{-1}\left(1 + \frac{\xi}{\sigma} I\right)^{-\frac{1}{\xi} - 1} \] (10),

With reparameterization, \( \lambda = \frac{1}{\xi} + 1 \) \( \phi = \frac{\sigma}{\xi} \), Eq. 10 becomes:

\[ f(I) = (\lambda - 1)\phi^{-1}(1 + \phi^{-1} I)^{-\lambda} \] (11).

For large values, Eq. 11 corresponds to a power law:

\[ (\lambda - 1)\phi^{-1}(1 + \phi^{-1} I)^{-\lambda} \rightarrow \left[(\lambda - 1)\phi^{\lambda - 1}\right]I^{-\lambda} \quad \text{for} \quad I \to \infty. \]

For small values, the generalized Pareto law prevents the singularity that causes the power law not to extend to zero: \( \lim_{I \to 0} f(I) = \infty \) in Eq. 2, while \( f(0) = (\lambda - 1)\phi^{-1} \) in Eq. 11.

In the case of a power law with \( \lambda \leq 2 \), \( \bar{I} \to \infty \) when \( I_{\text{max}} \to \infty \) in Eq. 2. Consequently, for these values of \( \lambda \), \( I_{\text{max}} \) should always be finite. The same rule applies to the generalized Pareto law. For \( \lambda \leq 2 \) one must use the truncated generalized Pareto law (TGPL):

\[ f(I) = \left[\frac{\lambda - 1}{\phi(1 + \phi^{-1} I_{\text{max}})^{-\lambda + 1}}\right](1 + \phi^{-1} I)^{-\lambda} \] (12),

From Eqs. 1 and 12, the mean of LPGT is:

\[ \bar{I} = \frac{\phi}{2 - \lambda}\left[\left(1 + \phi^{-1} I_{\text{max}}\right)^{-\lambda + 2} - 1\right] - I_{\text{max}}\left(1 + \phi^{-1} I_{\text{max}}\right)^{-\lambda + 1} \]

\[ \frac{1 - \left(1 + \phi^{-1} I_{\text{max}}\right)^{-\lambda + 1}}{\left(1 - \left(1 + \phi^{-1} I_{\text{max}}\right)^{-\lambda + 1}\right)} \] (13),

Eqs. 2 and 13 can be used to calculate the mean after fitting their respective distributions. However, any distribution is a simplification of reality. The sample mean is more robust since it does not presume a particular distribution. Dos Santos (2000) used his formula (Eq. 8) to calculate the emission considering only values within the observed limits. But for a calculation that is restricted to these limits, the most reliable method is the sample mean. However, if correctly applied, the power law (and its generalization in Eq. 11) is useful for taking a step forward and inferring the extreme events that are so infrequent that they are usually not observed during sampling (Pueyo 2007).

**REDOING THE CALCULATIONS**

To obtain an alternative to the official estimates, we applied different statistical distributions and compared the quality of the fits. The approach used was frequentist, like that of dos Santos (2000) and ELETROBRÁS (2000), so as not to divert attention to the frequentist/Bayesian debate and the details of the Bayesian approach. The results formed the basis for estimating mean emissions.

The parameters were adjusted by the maximum likelihood estimator (MLE), except for the upper cutoff point \( I_{\text{max}} \), for which the MLE has a systematic bias (Pueyo 2007). Since we do not have access to the original data, we developed a version of the MLE that starts from data that had been pre-grouped into bins (Appendix). The MLE was applied assuming the \( I_{\text{max}} \) value that we call "basic," which is the upper limit of the top bin. Afterwards, the "extrapolated" value of \( I_{\text{max}} \) was calculated by
the method of Pueyo (2007), corresponding to events that are so extreme that they are not included in the sample.

Distributions considered were the truncated power law (TPL), truncated generalized Pareto law (TGPL) and the negative exponential law (NEL), in addition to the TPL with parameters estimated by dos Santos (2000) (TPL-s) in place of the parameters estimated by MLE. The TGPL was studied because it is equivalent to TPL except at the lower end, where it is more realistic as was explained in the previous section. The NEL was studied to allow comparison with a distribution that is also asymmetric, but with events that are less extreme than in a power law (corresponding to the limit $\xi \to 0$ of Eq. 10):

$$f(I) = \sigma^{-1} e^{-I/\sigma}$$  \hspace{1cm} (14)

(initially, a truncated version of the NEL was considered so that it would be more comparable with the TPL, but this did not lead to any difference in the result because the estimated limits were $I_{\min} = 0$ and $I_{\max} \to \infty$). The four distributions were compared using the likelihood ratios.

Mean emissions were calculated for the TPL and the TGPL. As has been argued, the sample mean is probably the best estimator of the real mean for the “basic” $I_{\max}$ because it does not presuppose any concrete distribution. This cannot be calculated directly without access to the original data, but can be approximated with the equation:

$$\bar{I} = \sum_{j} n_{j} I_{j}$$  \hspace{1cm} (15),

where $I_{j}$ is an emission value representative of bin $j$ and $n_{j}$ is the number of data points in this bin. The most immediate option is to assign $I_{j}$ the central value of the bin (non-parametric method), but this represents an overestimate because, in the power law and similar distributions, the mean of the values within each bin is lower than the central value. To offset this bias, Eq. 15 was applied assigning to each $I_{j}$ the mean for the bin according to the TGPL (semi-parametric method). The sample mean reconstructed in this way was complemented with an estimation of undetected extreme events (again using the parameters of the TGPL). The Appendix gives more details on the methods.

The results of fitting the distributions are represented in Figures 1 and 2, which compare the empirical with the expected frequencies according to each hypothesis on a log-log scale.

[Figures 1 & 2 here]

In the case of bubbling, the exponent estimated by MLE was $\lambda = 1.21$ for the TPL and the TGPL, while dos Santos (2000) estimated $\lambda = 0.99$. Dos Santos assumed that $I_{\min} = 12.5$ mg m\(^{-2}\) d\(^{-1}\) while the value estimated by the MLE in the TPL is $I_{\min} = 0.53$ mg m\(^{-2}\) d\(^{-1}\). The value estimated for $\varphi$ (equivalent to $I_{\min}$) in the TGPL is $\varphi = 0.54$ mg m\(^{-2}\) d\(^{-1}\). The extrapolated upper limit is $I_{\max} = 596$ mg m\(^{-2}\) d\(^{-1}\) for the TPL and the TGPL.

For diffusion, the exponent estimated by MLE was $\lambda = 2.08$ for the TPL and $\lambda = 2.65$ for the TGPL, while dos Santos (2000) estimated $\lambda = 2.00$. Dos Santos took $I_{\min} = 12.5$ mg m\(^{-2}\) d\(^{-1}\), while the value estimated by MLE in $I_{\min}$ TPL is $7.99$ mg m\(^{-2}\) d\(^{-1}\). The value estimated for $\varphi$ in TGPL is $\varphi = 21.82$ mg m\(^{-2}\) d\(^{-1}\). The extrapolated upper limit is $I_{\max} = 714$ mg m\(^{-2}\) d\(^{-1}\) for TPL and $I_{\max} = 929$ mg m\(^{-2}\) d\(^{-1}\) for TGPL.

Tables I and II show the likelihood ratios between the different distributions. The TPL and the TGPL have almost equal likelihoods, many orders of magnitude above the likelihood of the TPL-s and the NEL.

[Tables I & II here]

Applying Eq. 9, dos Santos (2000) obtained mean emissions of 8.36 mg m\(^{-2}\) d\(^{-1}\) (bubbling) and 9.93 mg m\(^{-2}\) d\(^{-1}\) (diffusion). Applying the correct equation (Eq. 4) with the parameters that they
estimated, the means go up to 143 mg m$^{-2}$ d$^{-1}$ (bubbling) and 51.55 mg m$^{-2}$ d$^{-1}$ (diffusion). However, applying the same equation with the parameters of the TPL estimated by MLE (with the “basic” $I_{\text{max}}$), the results are 44.49 mg m$^{-2}$ d$^{-1}$ (bubbling) and 31.20 mg m$^{-2}$ d$^{-1}$ (diffusion). In the case of TGPL, the results are 44.33 mg m$^{-2}$ d$^{-1}$ (bubbling) and 27.26 mg m$^{-2}$ d$^{-1}$ (diffusion). Using the extrapolated $I_{\text{max}}$, the TPL gives 47.22 mg m$^{-2}$ d$^{-1}$ for bubbling and 32.71 mg m$^{-2}$ d$^{-1}$ for diffusion, while the TGPL gives 47.05 mg m$^{-2}$ d$^{-1}$ for bubbling and 29.03 mg m$^{-2}$ d$^{-1}$ for diffusion.

Applying the non-parametric method, the results are 54.57 mg m$^{-2}$ d$^{-1}$ for bubbling and 31.63 mg m$^{-2}$ d$^{-1}$ for diffusion. The semi-parametric method, which is probably the most realistic, gives 48.67 mg m$^{-2}$ d$^{-1}$ (bubbling) and 28.21 mg m$^{-2}$ d$^{-1}$ (diffusion), or 76.88 mg m$^{-2}$ d$^{-1}$ in total, or 10.9%, 10.7% and 10.8% lower, respectively. Considering extreme events that were not detected, the means rise moderately to 51.37 mg m$^{-2}$ d$^{-1}$ (bubbling), 29.97 mg m$^{-2}$ d$^{-1}$ (diffusion) and 81.34 mg m$^{-2}$ d$^{-1}$ (total), corresponding to increases of 5.5%, 5.2% and 5.8%, respectively.

These analyses support the idea that methane emissions are well characterized by a power law. The two variants of the power law we studied, TPL and TGPL, show the same performance, but this is a consequence of using data that were pre-grouped into bins. It is believed that the TGPL is best suited for data that are not grouped. All realistic approximations (TPL with parameters estimated by MLE and TGPL with parameters estimated by non-parametric and semi-parametric (MLE) give similar mean emission values between them and are clearly above the values estimated by dos Santos (2000) and ELETRÔBRAS (2000). While these studies estimated 8.36 mg m$^{-2}$ d$^{-1}$ for bubbling and 9.93 mg m$^{-2}$ d$^{-1}$ for diffusion, our best estimate is 51.37 mg m$^{-2}$ d$^{-1}$ for bubbling and 29.97 mg m$^{-2}$ d$^{-1}$ for diffusion (Fig. 3). Compared to the official estimates, our best estimate is 514% higher for bubbling, 202% higher for diffusion, and 345% higher for the two together, i.e., for the total emission from the reservoir surface (without considering the effect of the official figures having omitted emissions from the water that passes through the turbines and spillways).

COMPARISON OF RESULTS

Dos Santos et al. (2008) reported that the calculation they used resulted in a lower value than the simple arithmetic mean of the emission measurements, but did not mention the magnitude of this difference. Calculating emissions in both ways, the difference is huge (Fig. 3 and Table III). These differences exist both for CO$_2$ and for CH$_4$. Although ELETRÔBRAS (2000) and dos Santos et al. (2008) did calculations for both gases, we will only consider CH$_4$, which represents an impact on global warming, because, in the case of CO$_2$ emitted by the water in the reservoir, almost all of the emission comes from the decomposition of biotic material derived from the products of photosynthesis in or around the reservoir, which remove the same amount of CO$_2$ from the atmosphere. In contrast, the role of the reservoir in transforming CO$_2$ into CH$_4$ represents a net impact on global warming (with the exception of the small part that represents the action of the carbon that otherwise would have been emitted in the form of CO$_2$, this avoided impact being counted only over the roughly ten years that each molecule of CH$_4$ remains, on average, in the atmosphere).

The value of dos Santos et al. (2008) for emissions of CH$_4$ (adding bubbling and diffusion) is 78% lower than our best estimate (the mean corrected with adjustment by the power law: Table III), or expressed relative to the value of dos Santos et al. (2008), our value is 345% higher. The importance of this becomes evident when applied to the set of Brazilian dams, as was done by ELETRÔBRAS (2000) using the equations of dos Santos (2000). ELETRÔBRAS (2000) made this calculation for each of the 217 reservoirs in Brazil, totaling 32,975.48 km$^2$, an area larger than Belgium. When the emissions of CH$_4$ thus calculated are compared to emissions calculated by our best estimate (Table IV), the difference is 4.4 million tons of carbon equivalent to carbon in the form of CO$_2$ per year, when calculated using the global warming potential (GWP) of 21 for methane that the Kyoto Protocol used for the 2008-2012 period. The difference goes up to 5.2 million tons using the value 25 for the GWP, obtained from the most recent report of Intergovernmental Panel on
Climate Change (IPCC) (Forster et al. 2007). The difference goes up even more, to 7.0 million tons, when calculated with a GWP of 34, from a study done subsequent to the IPCC report, incorporating indirect effects that were omitted in previous calculations (Shindell et al. 2009). This difference is close to the annual emission by burning fossil fuels in greater São Paulo. São Paulo emitted 4.3 million tons of carbon in 2003 with a population of 10.7 million (COPPE 2005). Considering the entire metropolitan area today, the population is roughly twice that, presumably emitting around 8 million tons of carbon. The emission from the surfaces of reservoirs of hydroelectric dams in Brazil, considering our best estimate (81.4 mg m$^{-2}$ d$^{-1}$) and an up-to-date estimate of the GWP of methane (34), already exceeds this value, with 9.08 million tons of CO$_2$ carbon-equivalent per year.

Emissions from the surface of reservoirs are only part of the impact of hydroelectric dams on global warming, since the methane emission from water that passes through the turbines and spillways often more than doubles the total impact (e.g., Fearnside 2002, 2009).

CONCLUSION

The official figures for the emission of methane from the surfaces of Brazilian hydroelectric reservoirs have grossly underestimated these emissions. Our best estimate is more than three times higher. For the 33,000 km$^2$ of Brazilian reservoirs, the underestimate in the official numbers for the annual emissions of CH$_4$ by the surfaces of reservoirs is almost as large as the entire global-warming contribution made by fossil-fuel burning in greater São Paulo, and the corrected total for emissions from these reservoir surfaces surpasses the emission of this metropolis of over 20 million inhabitants. Emission from the water that passes through the spillways and turbines (not included in official numbers) represents a substantial additional impact.

APPENDIX

This Appendix adds some mathematical details of the methods used in the Redoing the calculations section.

To estimate the parameters, except for $I_{max}$, and to compare the hypotheses, the likelihood function ($L$) was used. This function is defined as:

$$L = f(I; \theta, H_d)$$

where $I$ is the vector of the data, $\theta$ is the vector of the parameters and $H_d$ is the hypothesis of the type of distribution. In this case the original data were not directly accessible, only being presented pre-grouped into $K$ bins. Therefore, the likelihood function used was:

$$L = f(n; \theta, H_d)$$

where $n$ is the vector ($n_1, ... , n_j, ... , n_K$) of the number of data points in bin $j$. In this case, $L$ represents a multinomial distribution,

$$L = \frac{N!}{n_1! ... n_j! ... n_K!} p_1^{n_1} ... p_j^{n_j} ... p_K^{n_K}$$

where $N$ is the total number of data points $N = \sum_{j=1}^{K} n_j$, and $p_j (\theta; H_d)$ is the expected probability of a data point belonging to bin $j$.

This probability has the following form for the truncated power law (TPL) (Eq. 2):

$$p_j = \frac{I_{max}^{\lambda+1} - c_j^{\lambda+1}}{I_{min}^{\lambda+1} - I_{max}^{\lambda+1}}$$

(19),
\[ p_j = \frac{c_j^{-\lambda+1} - c_{j-1}^{-\lambda+1}}{I_{\min}^{-\lambda+1} - I_{\max}^{-\lambda+1}}, \quad 1 < j \leq K \]  

(20),

where \( c_j \) is the upper limit of the bin \( j \) and we assume \( I_{\max} = c_K \) (following the criterion of the “basic” \( I_{\max} \), described below). For the truncated generalized Pareto law TGPL:

\[ p_j = \frac{\left(1 + \varphi^{-1}c_{j-1}\right)^{-\lambda+1} - \left(1 + \varphi^{-1}c_j\right)^{-\lambda+1}}{1 - \left(1 + \varphi^{-1}I_{\max}\right)^{-\lambda+1}} \]

(21).

For the negative exponential law (NEL):

\[ p_j = e^{-c_j/\sigma} - e^{-c_{j-1}/\sigma}, \quad 1 \leq j \leq K \]

(22).

The function \( L \) was used for two purposes. First, to fit the distributions by calculating the maximum likelihood estimator, which is the vector of parameters \( \theta \) that maximizes \( L \); in all cases, the parameters were fit numerically. Second, this function was used to compare the different hypotheses by using the likelihood ratio, which is obtained by dividing the \( L \) of one hypothesis by the \( L \) of the other hypothesis.

The only parameter that was not fit using the MLE method was \( I_{\max} \). The MLE of \( I_{\max} \) is the largest value observed, which is always an underestimate of the highest possible value. Our study considered two \( I_{\max} \) values. The first was the "basic" value, which was used to calculate \( L \). This consists of the upper limit of the top bin. After estimating the other parameters of the distribution, the "extrapolated" value of \( I_{\max} \) was calculated, which is an estimator of the true \( I_{\max} \) following the method of Pueyo (2007). In the case of the TPL,

\[ \hat{I}_{\max} = \left[I_{\min}^{-\lambda+1} - 2^{-\lambda} \left[I_{\min}^{-\lambda+1} - \text{max}(I)^{-\lambda+1}\right]\right]^{-1} \]

(23),

where max\((I)\) is the largest value in the sample. For pre-grouped data, the value assigned to max\((I)\) was the central value of the highest bin that contained data. In the case of the TGPL,

\[ \hat{I}_{\max} = \varphi \left[1 - 2^{-\lambda} \left[I_{\min}^{-\lambda+1} - \text{max}(I)^{-\lambda+1}\right]\right]^{-1} \]

(24).

The mean was estimated for the TPL and the TGPL applying Eqs. 4 and 13. It was also estimated by using a non-parametric and a semi-parametric method, based on Eq. 15. In the case of non-parametric method, the values of \( I_j \) correspond to the midpoint of each bin, \( I_j = (c_{j-1} + c_j)/2 \).

For the semi-parametric method, \( I_j \) corresponds to the expected mean of bin \( j \) according to the TGPL:

\[ I_j = \left(\frac{\varphi}{2^{\lambda} - \lambda}\right)\left[I_{\min}^{-\lambda+1} - \left(I + \varphi^{-1}c_{j-1}\right)^{-\lambda+1}\right] - c_{j-1} \left[1 + \varphi^{-1}c_j\right]^{-\lambda+1} + c_{j-1} \left[1 + \varphi^{-1}c_{j-1}\right]^{-\lambda+1} \]

(25).
ACKNOWLEDGEMENTS

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FEARNSIDE, P.M. 2009. As hidrelétricas de Belo Monte e Altamira (Babaquara) como fontes de gases de efeito estufa. Novos Cadernos NAEA, 12(2): 5-56.


Figure Legends

Figure 1. Statistical distribution of methane emissions from bubbling. The abscissas \((f)\) are the midpoints of the intervals, or bins, into which the data are grouped (in mg m\(^{-2}\) d\(^{-1}\)). The ordinates \((n)\) are the number of observations in each interval. Both axes have logarithmic scales. The open circles indicate the theoretical frequency of each distribution model. The solid circles indicate the empirical frequency. The solid triangles on the abscissa axes identify the intervals with zero observations. The goodness of fit is more difficult to appreciate for the intervals corresponding to the larger emissions, where the empirical frequencies are typically 0 or 1, while the theoretical frequencies of the models with a good fit are values that are intermediate between 0 and 1.

Figure 2. Statistical distribution of methane emissions by diffusion. See the caption of Figure 1 for details.

Figure 3. Comparison of the mean emissions of methane based on different criteria. The “ELETROBRÁS” criterion corresponds to dos Santos (2000) and ELETROBRÁS (2000) and is based on methodological errors. The “corrected” basic emission is the sample mean of the data used in these same studies (inferred from pre-binned data). The “corrected” extrapolated mean adds the probable effect of extreme events that were not included in the sample. The “corrected” extrapolated emission (our best estimate) exceeds the “ELETROBRÁS” value by 514% for bubbling, by 202% for diffusion, and by 345% for the sum of the two.
Table I. Likelihood ratios between the hypotheses considered (bubbling)\textsuperscript{a}.

<table>
<thead>
<tr>
<th></th>
<th>TPL-s</th>
<th>TPL</th>
<th>TGPL</th>
<th>NEL</th>
</tr>
</thead>
<tbody>
<tr>
<td>TPL-s</td>
<td>1.00</td>
<td>4.15 × 10\textsuperscript{31}</td>
<td>4.09 × 10\textsuperscript{31}</td>
<td>7.66 × 10\textsuperscript{9}</td>
</tr>
<tr>
<td>TPL</td>
<td>2.41 × 10\textsuperscript{-32}</td>
<td>1.00</td>
<td>0.99</td>
<td>1.85 × 10\textsuperscript{-22}</td>
</tr>
<tr>
<td>TGPL</td>
<td>2.44 × 10\textsuperscript{-32}</td>
<td>1.01</td>
<td>1.00</td>
<td>1.87 × 10\textsuperscript{-22}</td>
</tr>
<tr>
<td>NEL</td>
<td>1.31 × 10\textsuperscript{-10}</td>
<td>5.42 × 10\textsuperscript{21}</td>
<td>5.34 × 10\textsuperscript{21}</td>
<td>1.00</td>
</tr>
</tbody>
</table>

\textsuperscript{a}Values greater than 1 mean that the hypothesis in the column explains the data better than the hypothesis in the row, and vice versa. Details in the Appendix.
Table II. Likelihood ratios between the hypotheses considered (diffusion)\textsuperscript{a}.

<table>
<thead>
<tr>
<th></th>
<th>TPL-s</th>
<th>TPL</th>
<th>TGPL</th>
<th>NEL</th>
</tr>
</thead>
<tbody>
<tr>
<td>TPL-s</td>
<td>1.00</td>
<td>2.19 × 10\textsuperscript{11}</td>
<td>2.23 × 10\textsuperscript{11}</td>
<td>1.43 × 10\textsuperscript{-5}</td>
</tr>
<tr>
<td>TPL</td>
<td>4.57 × 10\textsuperscript{-12}</td>
<td>1.00</td>
<td>1.02</td>
<td>6.53 × 10\textsuperscript{-17}</td>
</tr>
<tr>
<td>TGPL</td>
<td>4.49 × 10\textsuperscript{-12}</td>
<td>0.98</td>
<td>1.00</td>
<td>6.41 × 10\textsuperscript{-17}</td>
</tr>
<tr>
<td>NEL</td>
<td>7.00 × 10\textsuperscript{4}</td>
<td>1.53 × 10\textsuperscript{16}</td>
<td>1.56 × 10\textsuperscript{16}</td>
<td>1.00</td>
</tr>
</tbody>
</table>

\textsuperscript{a}Values greater than 1 mean that the hypothesis in the column explains the data better than the hypothesis in the row, and vice-versa. Details in the Appendix.
Table III. Comparison of calculations of CH₄ emissions from hydroelectric reservoirs.

<table>
<thead>
<tr>
<th>Process</th>
<th>Values for emissions (I) in mg m⁻² d⁻¹</th>
<th>Percentage difference between results</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>“Basic” corrected mean</td>
<td>Corrected mean “extrapolated” with the power law</td>
</tr>
<tr>
<td>Bubbling</td>
<td>48.7</td>
<td>51.4</td>
</tr>
<tr>
<td>Diffusion</td>
<td>28.2</td>
<td>30.0</td>
</tr>
<tr>
<td>Total surface</td>
<td>76.9</td>
<td>81.4</td>
</tr>
</tbody>
</table>

*a Mean corrected with the semi-parametric method (see Appendix).
Table IV. Impact of CH₄ emission from the surface (bubbling + diffusion) of Brazilian reservoirs calculated by different methods considered in this study (corrected “basic” mean and the mean corrected with the power law), and by the methods of dos Santos et al. (2008).

<table>
<thead>
<tr>
<th></th>
<th>&quot;Basic&quot; corrected mean</th>
<th>Mean corrected with the power law</th>
<th>Dos Santos et al. (2008)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Emission (kg km⁻² d⁻¹)</td>
<td>76.9</td>
<td>81.3</td>
<td>18.29</td>
</tr>
<tr>
<td>Annual emission 32,975.48 km² (million tons)</td>
<td>0.93</td>
<td>0.98</td>
<td>0.22</td>
</tr>
<tr>
<td>CO₂-equivalent carbon⁸, using a GWP for CH₄ of 21⁹ (million tons year⁻¹)</td>
<td>5.30</td>
<td>5.61</td>
<td>1.26</td>
</tr>
<tr>
<td>CO₂-equivalent carbon⁸, using a GWP for CH₄ of 25¹ (million tons year⁻¹)</td>
<td>6.31</td>
<td>6.68</td>
<td>1.50</td>
</tr>
<tr>
<td>CO₂-equivalent carbon⁸, using a GWP for CH₄ of 34¹ (million tons year⁻¹)</td>
<td>8.59</td>
<td>9.08</td>
<td>2.04</td>
</tr>
</tbody>
</table>

⁸CO₂-equivalent carbon is calculated, by first multiplying the emission of CH₄ (in tons of gas) times the GWP (global warming potential) to obtain the equivalent number of tons of CO₂ gas, and then this is converted to the weight of carbon by multiplying by 12 (atomic weight of carbon) and dividing by 44 (molecular weight of CO₂). The GWP refers to the effect on global warming caused by the emission of one ton of gas (in this case CH₄) compared to the impact of one ton of CO₂. As in the Kyoto Protocol, the GWPs used have time horizons of 100 years with no application of a discount rate for time.


¹¹Forster et al. (2007).

¹²Shindell et al. (2009).
Fig. 1
Fig. 2
Fig. 3